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System Identification from Closed-Loop Data with Known Output Feedback Dynamics

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This paper formulates a method to identify the open-loop dynamics of a system operating under closed-loop conditions. The closed-loop excitation data and the feedback dynamics are assumed to be known. Two closed-loop configurations are considered where the system has either a linear output feedback controller or a dynamic output feedback controller. First, the closed-loop excitation data are used to compute the closed-loop Markov parameters, from which the open-loop Markov parameters are recovered from the known feedback dynamics. The Markov parameters are then used to compute a state space representation of the open-loop system. Examples are provided to illustrate the computational steps involved in the proposed closed-loop identification method.

Introduction

THE problem of identification of the open-loop system when it is operating under closed loop conditions is considered. This is a problem of considerable practical importance. Consider the case where a system is operating in closed loop, and it is not possible or desirable to remove the existing controller for open-loop identification. An open-loop model of the system may be needed for purpose of analysis or controller redesign. It is generally not possible to identify the open-loop system simply by measuring the system output and the actual input during closed-loop operation because such input is usually not rich enough for identification. Additive input excitation during closed-loop operation is necessary to identify the open-loop system. However, the excitation must be added in such a way that it does not affect overall system stability that is provided by the existing controller.

The paper considers the following closed-loop identification problems. First, the identification of a system having an existing linear output feedback controller is formulated. This is the simplest case, on which several extensions are made. Second, if the open-loop system has a direct transmission term, the mathematical problem becomes slightly more complicated, and this case is treated next. Third, the more general problem where a closed-loop system has output feedback dynamics is considered. Fourth, if the open-loop system includes known input and/or output filters in addition to the plant, it is sometimes possible to extract the plant dynamics from the combined dynamics. Mathematically, this is the case of recovering the dynamics of one system from the combined dynamics of two cascading linear systems when the other system is known. Numerical examples are also provided to illustrate the closed-loop identification techniques.

The identification method developed here is based on the concept of an observer for the closed-loop system. It can be viewed as an extension of the observer-based approach formulated in Refs. 1–4 for open-loop identification. Of importance are certain basic algebraic relations between the Markov parameters of the observer and those of the closed-loop and open-loop systems. A review of the Markov parameters in system identification is presented in Ref. 5. From the Markov parameters, a state space realization of the system can be computed as shown in Ref. 6. In this paper, the output feedback dynamics is assumed known. This requirement makes the problem mathematically well-posed. If feedback dynamics is not known, then the feedback signal must be known, which is the case treated in Ref. 7 for a closed-loop system possessing a full state feedback structure.

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Problem Statement

Consider a linear multivariable system expressed in state space format as

$$
\begin{aligned}
x(i+1) &= Ax(i) + Bu(i) \\
y(i) &= Cx(i)
\end{aligned}
\tag{1}
$$

where $x(i) \in R^n$, $y(i) \in R^q$, and $u(i) \in R^m$. The system has an existing linear output feedback controller with a gain F. For purpose of identification, an additional excitation $v(i)$ is injected to the control input, and the total input to the closed-loop system becomes

$$
u(i) = Fy(i) + v(i) \tag{2}
$$

which yields the following closed-loop system of the form

$$
\begin{aligned}
x(i+1) &= A_c x(i) + Bv(i) \\
y(i) &= Cx(i)
\end{aligned}
\tag{3}
$$

where A_c denotes the closed-loop system matrix, $A_c = A + BFC$. The excitation input data $v(i)$, the corresponding closed-loop response $y(i)$, and the existing linear feedback gain F are assumed to be known (see Fig. 1). The objective of the problem is to obtain a state space model of the open-loop system, denoted by the set A, B, C from the known closed-loop data. Furthermore, this is to be accomplished by first obtaining the Markov parameters $Y(i) = CA^{i-1}B$, $i = 1, 2, \ldots,$ from which the matrices A, B, C are realized. Subsequent developments that treat several extensions of this basic problem will be considered.

Mathematical Formulation

The mathematical formulation consists of the following developments. The input-output description of the closed-loop system is described in terms of an observer whose Markov parameters can be computed from closed-loop data. Algebraic relations between the Markov parameters of the observer and those of the closed-loop and open-loop system are derived. From these relations, the open-loop system dynamics can be recovered from the computed observer Markov parameters and knowledge of the feedback gain. For clarity of presentation, the formulation is done first for the case without a direct transmission term, then extended to the case with a direct transmission term. This is because inclusion of the transmission term at the beginning would complicate the algebra without altering it fundamentally. Also, the case of the closed-loop system with a linear output feedback controller is treated first. The case with a dynamic controller then follows. Further extensions to the situation when the open-loop plant includes input or output filters are considered.

Closed-Loop System and Its Associated Observer

This section introduces the concept of an observer as an intermediate step in solving for the Markov parameters of the open-loop system. First, note from Eq. (3) that the additive excitation signal $v(i)$ does not affect the overall closed-loop system stability that is being provided by the feedback controller since it does not alter the existing closed loop system matrix $A_c = A + BFC$. To solve for the Markov parameters of the closed-loop system in Eq. (3), an observer is introduced to this set of equations by adding and subtracting the term $My(i)$ to the right-hand side of the state equation in Eq. (3),

$$\begin{aligned} x(i+1) &= A_c x(i) + Bv(i) + My(i) - My(i) \\ &= (A_c + MC)x(i) + Bv(i) - My(i) \end{aligned} \tag{4}$$

Define the following quantities:

$$\bar{A}_c = A_c + MC, \qquad \bar{B} = [B, -M], \qquad z(i) = \begin{bmatrix} v(i) \\ y(i) \end{bmatrix} \tag{5}$$

Then the original closed-loop system can be expressed as

$$\begin{aligned} x(i+1) &= \bar{A}_c x(i) + \bar{B}z(i) \\ y(i) &= Cx(i) \end{aligned} \tag{6}$$

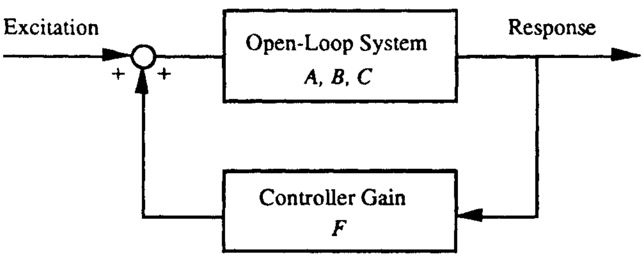

Fig. 1 Block diagram of the closed-loop system.

This operation is equivalent to introducing an observer to the closed-loop system if the state $x(i)$ is considered as an observer state and the matrix M an observer gain. The freedom introduced by M will be used to influence the stability of $\bar{A}_c$. Specifically, the matrix M is specified implicitly through the following dead-beat condition:

$$\bar{A}_c^p = (A_c + MC)^p \equiv 0 \tag{7}$$

Then the input-output relation can be described exactly by

$$y(i) = \sum_{\tau=1}^{p} \bar{Y}_c(\tau) z(i-\tau) \tag{8}$$

where the (closed-loop) observer Markov parameters $\bar{Y}_c(i)$, $i = 1, 2, \ldots, p$, are defined as $\bar{Y}_c(i) = C\bar{A}_c^{i-1}\bar{B}$ and $\bar{Y}_c(i) = 0$ for $i = p+1, p+2, \ldots$.

Suppose that a set of N measurements of the closed-loop system $y(i)$ and $u(i)$, $i = 0, 2, \ldots, N-1$, given in Eq. (3) is available. Assuming zero initial conditions for the moment, the input output relation for the set of measurements can be written in matrix form as

$$y = \bar{Y}_c V \tag{9}$$

where

$$y = [y(1) \quad y(2) \quad \cdots \quad y(p) \quad y(p+1) \quad \cdots \quad y(N-1)]$$

$$\bar{Y}_c = [C\bar{B} \quad C\bar{A}_c\bar{B} \quad \cdots \quad C\bar{A}_c^{p-1}\bar{B}]$$

$$V = \begin{bmatrix} z(0) & z(1) & \cdots & z(p-1) & z(p) & \cdots & z(N-2) \\ & z(0) & \cdots & z(p-2) & z(p-1) & \cdots & z(N-3) \\ & & \ddots & \vdots & \vdots & & \vdots \\ & & & z(0) & z(1) & \cdots & z(N-p-1) \end{bmatrix}$$

From Eq. (9) the set of observer Markov parameters of the closed-loop system can be solved for provided that the additive excitation is sufficiently rich such that the $v(i)$ rows in V have full rank. The least-squares solution to the observer Markov parameter matrix $\bar{Y}_c$ is given as

$$\bar{Y}_c = yV^+ \tag{10}$$

where $(\cdot)^+$ denotes the pseudoinverse of the quantity in the parentheses. For nonzero initial conditions a somewhat different equation must be used. The appropriate replacement equation for Eq. (9) is simply

$$y_t = \bar{Y}_c V_t \tag{11}$$

where the truncated output matrix y_t and the truncated input matrix V_t are

$$y_t = [y(p) \quad y(p+1) \quad \cdots \quad y(N-1)]$$

$$V_t = \begin{bmatrix} z(p-1) & z(p) & \cdots & z(N-2) \\ z(p-2) & z(p-1) & \cdots & z(N-3) \\ \vdots & \vdots & & \vdots \\ z(0) & z(1) & \cdots & z(N-p-1) \end{bmatrix}$$

and the solution for $\bar{Y}_c$ is

$$\bar{Y}_c = y_t V_t^+ \tag{12}$$

The quantities y_t and V_t are obtained from y and V by simply deleting their first $p-1$ columns, respectively. This makes use of the fact that $\bar{A}^i \equiv 0$ for $i \geq p$, and the effect of the nonzero initial conditions can be neglected after p time steps.

Similar to the analysis in Ref. 2, the number of observer Markov parameters characterized by the integer p that are to be identified must be chosen such that $pq \geq n$, where n is the order of the system and q is the number of outputs. If p observer Markov parameters are identified, then the maximum order of a system that can be realized is pq.

Relationship Between Observer Markov Parameters and Closed-Loop System Markov Parameters

The previous section shows that the observer Markov parameters of the closed-loop system can be computed from closed-loop data. This section now shows that the Markov parameters of the closed-loop system can be recovered from those of the closed-loop observer.

Define $\bar{Y}_c(i) = [\bar{Y}_c^{(1)}(i)\ \bar{Y}_c^{(2)}(i)]$, where $\bar{Y}_c^{(1)}(i)$ and $\bar{Y}_c^{(2)}(i)$ are of dimensions $q \times m$ and $m \times m$, respectively. The Markov parameters of the closed-loop system defined as

$$Y_c(i) = CA_c^{i-1}B = C(A + BFC)^{i-1}B \tag{13}$$

can be recovered from the observer Markov parameters $\bar{Y}_c(i)$ by the following relation:

$$Y_c(i) = \bar{Y}_c^{(1)}(i) + \sum_{\tau=1}^{i-1} \bar{Y}_c^{(2)}(\tau)Y_c(i-\tau) \tag{14}$$

where $\bar{Y}_c(i) \equiv 0$ for $i \geq p$. In matrix form, the preceding recursive equation can be written as

$$\begin{bmatrix} I & & & & \\ \bar{Y}_c^{(2)}(1) & I & & & \\ \bar{Y}_c^{(2)}(2) & \bar{Y}_c^{(2)}(1) & I & & \\ \vdots & \vdots & \ddots & \ddots & \\ \bar{Y}_c^{(2)}(i-1) & \bar{Y}_c^{(2)}(i-2) & \cdots & \bar{Y}_c^{(2)}(1) & I \end{bmatrix} \begin{bmatrix} Y_c(1) \\ Y_c(2) \\ Y_c(3) \\ \vdots \\ Y_c(i) \end{bmatrix} = \begin{bmatrix} \bar{Y}_c^{(1)}(1) \\ \bar{Y}_c^{(1)}(2) \\ \bar{Y}_c^{(1)}(3) \\ \vdots \\ \bar{Y}_c^{(1)}(i) \end{bmatrix}$$

The matrix on the left-hand side of the equation just given is square and full rank, thus for a given set of observer Markov parameters $\bar{Y}_c(i)$, the closed-loop system Markov parameters $Y_c(i)$ can be uniquely determined.

Relationship Between Open-Loop and Closed-Loop System Markov Parameters with a Linear Output Feedback Controller

Recall that the main purpose of the problem is to compute the Markov parameters of the open-loop system. This section will show that the open-loop Markov parameters can be recovered from the closed-loop Markov parameters and knowledge of the feedback gain.

First, the first few Markov parameters of the open-loop system can be easily derived from those of the closed-loop system,

$$Y(1) = Y_c(1) = CB$$
$$Y(2) = Y_c(2) - Y_c(1)FY(1)$$
$$Y(3) = Y_c(3) - Y_c(1)FY(2) - Y_c(2)FY(1)$$

By induction, the relationship between the Markov parameters of the closed-loop system given in Eq. (3) and those of the open-loop system given in Eq. (1) is a weighted convolution sum, with the weighting matrix being the closed-loop gain F. In general, this expression is written as

$$Y(i) = Y_c(i) - \sum_{\tau=1}^{i-1} Y_c(\tau)FY(i-\tau) \tag{15}$$

for $i = 1, 2, 3, \ldots$. The preceding recursive equation can be written in matrix form as

$$\begin{bmatrix} I & & & & \\ Y_c(1)F & I & & & \\ Y_c(2)F & Y_c(1)F & I & & \\ \vdots & \vdots & \ddots & \ddots & \\ Y_c(i-1)F & Y_c(i-2)F & \cdots & Y_c(1)F & I \end{bmatrix} \begin{bmatrix} Y(1) \\ Y(2) \\ Y(3) \\ \vdots \\ Y(i) \end{bmatrix} = \begin{bmatrix} Y_c(1) \\ Y_c(2) \\ Y_c(3) \\ \vdots \\ Y_c(i) \end{bmatrix}$$

Each product $Y_c(i)F$ is a $q \times q$ square matrix. The left-hand-side matrix in the equation is square and full rank. This implies that the open-loop system Markov parameters can be uniquely recovered from the closed-loop system Markov parameters by this method.

Closed-Loop Identification of System with Direct Transmission Term

In the basic problem considered in the previous sections, the system description does not contain a direct transmission term. In identification and control of flexible structures, accelerometers are often used as measurement sensors, which will introduce a direct transmission term in the state space model. Identification of such systems in the closed loop is slightly more complicated. Consider a state space model with a direct transmission term D,

$$\begin{aligned} x(i+1) &= Ax(i) + Bu(i) \\ y(i) &= Cx(i) + Du(i) \end{aligned} \tag{16}$$

where again $x(i) \in R^n$, $y(i) \in R^q$, and $u(i) \in R^m$. The input equation given in Eq. (2) in this case is

$$\begin{aligned} u(i) &= Fy(i) + v(i) \\ &= FCx(i) + FDu(i) + v(i) \end{aligned} \tag{17}$$

The closed-loop system dynamics can be derived as follows. First, from Eq. (17), the controller input can be expressed as

$$u(i) = (I - FD)^{-1}FCx(i) + (I - FD)^{-1}v(i) \tag{18}$$

provided that the inverse $(I - FD)^{-1}$ exists. The set of closed-loop dynamics equations describing the relationship between the excitation $v(i)$ and output $y(i)$ is

$$\begin{aligned} x(i+1) &= [A + B(I - FD)^{-1}FC]x(i) + B(I - FD)^{-1}v(i) \\ y(i) &= (I - DF)^{-1}Cx(i) + (I - DF)^{-1}Dv(i) \end{aligned} \tag{19}$$

provided that the inverse $(I - DF)^{-1}$ exists. To derive the observer equations for the given system, add and subtract $My(i)$ to the right-hand side of the state equation in Eq. (19) to obtain

$$\begin{aligned} x(i+1) &= \bar{A}_c x(i) + \bar{B}_c z(i) \\ y(i) &= Cx(i) + Du(i) \end{aligned} \tag{20}$$

where

$$\bar{A}_c = A_c + MC, \qquad A_c = A + B_c FC, \qquad B_c = B(I - FD)^{-1}$$

$$\bar{B}_c = [B_c + MD - M(I - DF)], \qquad z(i) = \begin{bmatrix} v(i) \\ y(i) \end{bmatrix}$$

Let M be a deadbeat observer gain, $\bar{A}_c^p = (A_c + MC)^p \equiv 0$, then the input-output relation can be expressed as

$$y(i) = \sum_{\tau=1}^{p} \bar{Y}_c(\tau) z(i - \tau) + Du(i) \tag{21}$$

where $\bar{Y}_c(i) = C\bar{A}_c^{i-1}\bar{B}_c$. Writing in matrix form, the closed-loop observer Markov parameters are related to set of input-output data by the following equation:

$$y = \bar{Y}_c V \tag{22}$$

where

$$y = [y(0) \quad y(1) \quad y(2) \quad \cdots \quad y(p) \quad y(p+1) \quad \cdots \quad y(N-1)]$$

$$\bar{Y}_c = [D \quad C\bar{B}_c \quad C\bar{A}_c\bar{B}_c \quad \cdots \quad C\bar{A}_c^{p-1}\bar{B}_c]$$

$$V = \begin{bmatrix} u(0) & u(1) & u(2) & \cdots & u(p) & u(p+1) & \cdots & u(N-1) \\ & z(0) & z(1) & \cdots & z(p-1) & z(p) & \cdots & z(N-2) \\ & & z(0) & \cdots & z(p-2) & z(p-1) & \cdots & z(N-3) \\ & & & \ddots & \vdots & \vdots & & \vdots \\ & & & & z(0) & z(1) & \cdots & z(N-p-1) \end{bmatrix}$$

Equation (22) can be used to solve for the closed-loop observer Markov parameters. The solution is given in Eq. (10), with the exception that the output and input matrices are now given, respectively. If the initial conditions are not zero, then the truncated version of the corresponding output and input matrices are

$$y_t = [y(p) \quad y(p+1) \cdots y(N-1)]$$

$$V_t = \begin{bmatrix} u(p) & u(p+1) & \cdots & u(N-1) \\ z(p-1) & z(p) & \cdots & z(N-2) \\ z(p-2) & z(p-1) & \cdots & z(N-3) \\ \vdots & \vdots & & \vdots \\ z(0) & z(1) & \cdots & z(N-p-1) \end{bmatrix}$$

Again, to be able to solve for the observer Markov parameter matrix $\bar{Y}_c$, the rows of $v(i)$ in $z(i)$ must be full rank. Also, the number of observer Markov parameters characterized by the integer p that are to be identified must be chosen such that $pq \geq n$.

The next step is to show that the open-loop system Markov parameters $Y(i) = CA^{i-1}B$ can be recovered from the identified observer Markov parameters

$$\begin{aligned} \bar{Y}_c(i) &= C\bar{A}_c^{i-1}\bar{B}_c \\ &= [C\bar{A}_c^{i-1}(B_c + MD), \\ &\quad - C\bar{A}_c^{i-1}M(I - DF)], \\ &\quad i = 1, 2, \ldots, p \end{aligned}$$

and $\bar{Y}_c(i) = 0$, $i = p + 1, p + 2, \ldots$. To this end, define the first and second partitions of $\bar{Y}_c(i)$ as

$$\begin{aligned} \bar{Y}_c^{(1)}(i) &= C\bar{A}_c^{i-1}(B_c + MD) \\ &= C[A + B(I - FD)^{-1} FC + MC]^{i-1} \\ &\quad [B(I - FD)^{-1} + MD] \\ \bar{Y}_c^{(2)}(i) &= -C\bar{A}_c^{i-1}M(I - DF) \\ &= -C[A + B(I - FD)^{-1}FC + MC]^{i-1} \\ &\quad M(I - DF) \end{aligned}$$

From $\bar{Y}_c(i)$ the Markov parameters sequences $Y_c(i) = CA_c^{i-1}B_c$, $i = 1, 2, \ldots, p, p + 1, \ldots$, can be computed from the following recursive equation:

$$Y_c(i) = \bar{Y}_c^{(1)}(i) + \sum_{\tau=1}^{i} \bar{Y}_c^{(2)}(\tau)(I - DF)^{-1}Y_c(i - \tau) \tag{23}$$

where $Y_c(0) = \bar{Y}_c(0) = D$ and $\bar{Y}_c(i) = 0$, $i = p + 1, p + 2, \ldots$. Finally, the Markov parameters $Y_c(i) = C(A + B_cFC)^{i-1}B_c$ computed earlier have the same structure as the Markov parameters $Y_c(i)$ given in Eq. (13), where $B_c = B(I - FD)^{-1}$ now plays the role of B. Therefore, the open-loop system Markov parameters $Y(i) = CA^{i-1}B$ can be computed from $Y_c(i) = C(A + B_cFC)^{i-1}B_c$ as

$$Y(i) = Y_c(i)(I - FD) - \sum_{\tau=1}^{i-1} Y_c(\tau)FY(i - \tau) \tag{24}$$

Closed-Loop Identification of System with a Dynamic Feedback Controller

In this section, the identification of the open-loop model from a closed-loop system having an existing dynamic feedback controller is considered. Consider again a system in state space representation as in Eq. (16). The system has a dynamic feedback controller which for closed-loop identification is excited according to

$$\begin{aligned} s(i+1) &= Ps(i) + Qy(i) + v_2(i) \\ u(i) &= Rs(i) + Sy(i) + v_1(i) \end{aligned} \tag{25}$$

where $s(i) \in R^{n_s}$, $u(i) \in R^m$, and $y(i) \in R^q$. The scalar n_s denotes the order of the dynamic controller $n_s \leq n$. The vector quantities $v_1(i)$ and $v_2(i)$ denote the additive excitation signals for closed-loop identification. First, the closed-loop dynamics is derived. The input to the system is simply

$$u(i) = (I - SD)^{-1}[Rs(i) + SCx(i) + v_1(i)] \tag{26}$$

provided that the inverse $(I - SD)^{-1}$ exists. It can be shown that the closed-loop dynamics is governed by the following set of equations:

$$\begin{aligned} \begin{bmatrix} x(i+1) \\ s(i+1) \end{bmatrix} &= \begin{bmatrix} A + B(I - SD)^{-1}SC & B(I - SD)^{-1}R \\ QC + QD(I - SD)^{-1}SC & P + QD(I - SD)^{-1}R \end{bmatrix} \\ &\quad \times \begin{bmatrix} x(i) \\ s(i) \end{bmatrix} + \begin{bmatrix} B(I - SD)^{-1} & 0 \\ QD(I - SD)^{-1} & I \end{bmatrix} \begin{bmatrix} v_1(i) \\ v_2(i) \end{bmatrix} \end{aligned}$$

Furthermore,

$$\begin{bmatrix} y(i) \\ s(i) \end{bmatrix} = \begin{bmatrix} C & 0 \\ 0 & I_{n_s} \end{bmatrix} \begin{bmatrix} x(i) \\ s(i) \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u(i) \\ s(i+1) \end{bmatrix}$$

$$\begin{bmatrix} u(i) \\ s(i+1) \end{bmatrix} = \begin{bmatrix} S & R \\ Q & P \end{bmatrix} \begin{bmatrix} y(i) \\ s(i) \end{bmatrix} + \begin{bmatrix} v_1(i) \\ v_2(i) \end{bmatrix}$$

To reveal the mathematical structure of the preceding equations, define the following augmented system characterized by the matrices as in Ref. 8:

$$
\begin{aligned}
A_a &= \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \qquad B_a = \begin{bmatrix} B & 0 \\ 0 & I_{n_s} \end{bmatrix} \\
C_a &= \begin{bmatrix} C & 0 \\ 0 & I_{n_s} \end{bmatrix}, \qquad D_a = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \\
F_a &= \begin{bmatrix} S & R \\ Q & P \end{bmatrix}
\end{aligned} \tag{27}
$$

and the following augmented state, input, output, and excitation vectors

$$
\begin{aligned}
x_a(i) &= \begin{bmatrix} x(i) \\ s(i) \end{bmatrix}, \qquad u_a(i) = \begin{bmatrix} u(i) \\ s(i+1) \end{bmatrix} \\
y_a(i) &= \begin{bmatrix} y(i) \\ s(i) \end{bmatrix}, \qquad v_a(i) = \begin{bmatrix} v_1(i) \\ v_2(i) \end{bmatrix}
\end{aligned} \tag{28}
$$

Furthermore, since

$$
\begin{bmatrix} (I-SD)^{-1} & 0 \\ QD(I-SD)^{-1} & I \end{bmatrix} = \begin{bmatrix} I-DS & 0 \\ -QD & I \end{bmatrix}^{-1} = (I - F_aD_a)^{-1}
$$

It can be verified by direct substitution that

$$
\begin{bmatrix} A + B(I-SD)^{-1}SC & B(I-SD)^{-1}R \\ QC + QD(I-SD)^{-1}SC & P + QD(I-SD)^{-1}R \end{bmatrix} = A_a + B_a(I - F_aD_a)^{-1}F_aC_a
$$

$$
\begin{bmatrix} B(I-SD)^{-1} & 0 \\ QD(I-SD)^{-1} & I \end{bmatrix} = B_a(I - F_aD_a)^{-1}
$$

Using these identities, the set of equations describing closed-loop dynamics becomes

$$
\begin{aligned}
x_a(i+1) &= [A_a + B_a(I - F_aD_a)^{-1}F_aC_a]x_a(i) \\
&\quad + B_a(I - F_aD_a)^{-1}v_a(i) \\
y_a(i) &= (I - D_aF_a)^{-1}C_ax_a(i) + (I - D_aF_a)^{-1}D_av_a(i)
\end{aligned} \tag{29}
$$

provided that the inverse $(I - D_aF_a)^{-1}$ exists. As before, the additive excitation does not affect the overall system stability that is provided by the dynamic feedback controller. Comparing Eq. (29) to Eq. (19) reveals that the augmented system for the closed-loop dynamic controller has the same form as the following system:

$$
\begin{aligned}
x_a(i+1) &= A_ax_a(i) + B_au_a(i) \\
y_a(i) &= C_ax_a(i) + D_au_a(i)
\end{aligned} \tag{30}
$$

with a feedback controller

$$
u_a(i) = F_ay_a(i) + v_a(i) \tag{31}
$$

that includes an excitation term $v_a(i)$. At this point, results derived for the direct output feedback case apply. The observer equations for the augmented system can be derived by adding and subtracting the term $M_ay_a(i)$ to the augmented state equation in Eq. (29). Further algebraic manipulation produces the following set of observer equations for the augmented system:

$$
\begin{aligned}
x_a(i+1) &= \bar{A}_{ac}x_a(i) + \bar{B}_{ac}z(i) \\
y_a(i) &= C_ax_a(i) + D_au_a(i)
\end{aligned} \tag{32}
$$

where

$$
\bar{A}_{ac} = A_{ac} + M_aC_a, \qquad A_{ac} = A_a + B_{ac}F_aC_a
$$

$$
B_{ac} = B_a(I - F_aD_a)^{-1}
$$

$$
\bar{B}_{ac} = [B_{ac} + M_aD_a - M_a(I - D_aF_a)], \qquad z_a(i) = \begin{bmatrix} v_a(i) \\ y_a(i) \end{bmatrix}
$$

The deadbeat condition, $\bar{A}_{ac}^p = (A_{ac} + M_aC_a)^p \equiv 0$, is imposed on the gain M_a. The observer Markov parameters for the system given in Eq. (30) are

$$
\begin{aligned}
&\bar{Y}_{ac}(0) = D_a \\
&\bar{Y}_{ac}(i) = C_a\bar{A}_{ac}^{i-1}\bar{B}_{ac} = [C_a\bar{A}_{ac}^{i-1}(B_{ac} + M_aD_a), \\
&\qquad -C_a\bar{A}_{ac}^{i-1}M_a(I - D_aF_a)] \\
&i = 1, 2, \dots, p \qquad \bar{Y}_{ac}(i) \equiv 0, \qquad i = p+1, p+2, \dots
\end{aligned} \tag{33}
$$

Assuming zero initial conditions, the observer Markov parameters can be solved from

$$
\bar{Y}_{ac} = y_a(V_a)^+ \tag{34}
$$

where

$$
y_a = [y_a(0)\; y_a(1)\; y_a(2)\; \cdots\; y_a(p)\; y_a(p+1)\; \cdots\; y_a(N-1)]
$$

$$
\bar{Y}_{ac} = [D_a \quad C_a\bar{B}_{ac} \quad C_a\bar{A}_{ac}\bar{B}_{ac} \quad \cdots \quad C_a\bar{A}_{ac}^{p-1}\bar{B}_{ac}]
$$

$$
V_a = \begin{bmatrix}
u_a(0) & u_a(1) & u_a(2) & \cdots & u_a(p) & u_a(p+1) & \cdots & u_a(N-1) \\
 & z_a(0) & z_a(1) & \cdots & z_a(p-1) & z_a(p) & \cdots & z_a(N-2) \\
 & & z_a(0) & \cdots & z_a(p-2) & z_a(p-1) & \cdots & z_a(N-3) \\
 & & & \ddots & \vdots & \vdots & & \vdots \\
 & & & & z_a(0) & z_a(1) & \cdots & z_a(N-p-1)
\end{bmatrix}
$$

If the initial conditions are not zero, then the truncated versions of y_a and V_a are to be used. They are obtained by simply deleting the first p columns of y_a and V_a, respectively. In this case, the number of observer Markov parameters characterized by the integer p that are to be identified must be chosen such that $p(q + n_s) \geq n + n_s$.

As before, define the first and second partitions of $\overline{Y}_{ac}(i)$ as

$$\begin{aligned}\overline{Y}^{(1)}_{ac}(i) &= C_a\overline{A}_{ac}^{i-1}(B_{ac} + M_aD_a)\\ &= C_a[A_a + B_a(I - F_aD_a)^{-1}F_aC_a + M_aC_a]^{i-1}\\ &\quad [B_a(I - F_aD_a)^{-1} + M_aD_a]\end{aligned}$$

$$\begin{aligned}\overline{Y}^{(2)}_{ac}(i) &= -C_a\overline{A}_{ac}^{i-1}M_a(I - D_aF_a)\\ &= -C_a[A_a + B_a(I - F_aD_a)^{-1}F_aC_a + M_aC_a]^{i-1}\\ &\quad M_a(I - D_aF_a)\end{aligned}$$

The recursive equations for the Markov parameters $Y_{ac}(i) = C_aA_{ac}^{i-1}B_a$ and $Y_a(i) = C_aA_a^{i-1}B_a$ for $i = 1, 2, \ldots, p, p+1, \ldots,$ are

$$Y_{ac}(i) = \overline{Y}^{(1)}_{ac}(i) + \sum_{\tau=1}^{i} \overline{Y}^{(2)}_{ac}(\tau)(I - D_aF_a)^{-1}Y_{ac}(i - \tau) \tag{35}$$

$$Y_a(i) = Y_{ac}(i)(I - F_aD_a) - \sum_{\tau=1}^{i-1} Y_{ac}(\tau)F_aY_a(i - \tau) \tag{36}$$

where $\overline{Y}_{ac}(0) = Y_{ac}(0) = D_a$, and $\overline{Y}_{ac}(i) = 0$ for $i = p + 1, p + 2, \ldots$. Finally, it remains to be shown that the open-loop system Markov parameters $Y(0) = D$, $Y(i) = CA^{i-1}B$, $i = 1, 2, \ldots$, can be recovered from the Markov parameters $Y_a(i)$. This is indeed possible, since

$$Y_a(0) = \begin{bmatrix} Y(0) & 0\\ 0 & 0\end{bmatrix}, \qquad Y_a(1) = \begin{bmatrix} Y(1) & 0\\ 0 & I\end{bmatrix}$$

$$Y_a(i) = \begin{bmatrix} Y(i) & 0\\ 0 & 0\end{bmatrix}, \qquad i = 2, 3, \ldots$$

Thus, the open-loop system Markov parameters appear in the upper left partitions of the computed Markov parameters $Y_a(i)$, $i = 0, 1, 2, \ldots$.

Identification from Combined Markov Parameters

In practice, the open-loop system considered in previous sections may actually include the plant and the input/output filters. If the dynamics of the open-loop system is known and the filter dynamics is known, then under certain conditions, it is possible to recover the dynamics of the plant alone. Mathematically, this is the case of a combined system consisting of two cascading linear systems, and the problem is to compute the Markov parameters of one system if the Markov parameters of the other system and those of the combined system are known. First, the relationship between the Markov parameters of the combined system and those of the individual systems need to be derived. Let one system denoted by

$$\begin{aligned}x_1(i + 1) &= A_1x_1(i) + B_1u_1(i)\\ y_1(i) &= C_1x_1(i) + D_1u_1(i)\end{aligned} \tag{37}$$

be preceded by another system

$$\begin{aligned}x_2(i + 1) &= A_2x_2(i) + B_2u_2(i)\\ y_2(i) &= C_2x_2(i) + D_2u_2(i)\end{aligned} \tag{38}$$

where $x_2(i) \in R^{n_1}$, $y_1(i) \in R^{q_1}$, and $u_1(i) \in R^{m_1}$; and $x_2(i) \in R^{n_2}$, $y_2(i) \in R^{q_2}$, and $u_2(i) \in R^{m_2}$. Let the output of the second system be the input to the first system, i.e., $y_2(i) = u_1(i)$, $q_2 = m_1$. Then the combined system dynamics is given as

$$\begin{aligned}x_t(i + 1) &= A_tx_t(i) + B_tu_2(i)\\ y_1(i) &= C_tx_t(i) + D_tu_2(i)\end{aligned} \tag{39}$$

where

$$\begin{gathered}A_t = \begin{bmatrix} A_1 & B_1C_2\\ 0 & A_2\end{bmatrix}, \quad B_t = \begin{bmatrix} B_1D_2\\ B_2\end{bmatrix}, \quad C_t = [C_1 \quad D_1C_2]\\ D_t = D_1D_2, \qquad x_t(i) = \begin{bmatrix} x_1(i)\\ x_2(i)\end{bmatrix}\end{gathered} \tag{40}$$

The relationship between the Markov parameters of the combined system and those of the individual systems is established by noting that

$$\begin{aligned}Y_t(0) &= D_t = Y_1(0)Y_2(0)\\ Y_t(1) &= C_tB_t = Y_1(1)Y_2(0) + Y_1(0)Y_2(1)\\ Y_t(2) &= C_tA_tB_t = Y_1(2)Y_2(0) + Y_1(1)Y_2(1) + Y_1(0)Y_2(2)\end{aligned}$$

By induction, the general relationship is simply

$$Y_t(i) = \sum_{\tau=0}^{i} Y_1(\tau)Y_2(i - \tau) \tag{41}$$

Using this relationship, under certain conditions, it is possible to solve for the Markov parameters of one system when the Markov parameters of the combined system and of the other system are known. In particular, the following cases apply.

If the Markov parameters of the combined system and of system 1 are known, then the Markov parameters of system 2 can be solved using the following equations:

$$Y_2(0) = [Y_1^T(0)Y_1(0)]^{-1}Y_1^T(0)Y_t(0) \tag{42}$$

$$Y_2(i) = [Y_1^T(0)Y_1(0)]^{-1}Y_1^T(0)\left[Y_t(i) - \sum_{\tau=0}^{i-1} Y_1(\tau)Y_2(i - \tau)\right] \qquad i = 1, 2, \ldots \tag{43}$$

provided $q_1 \geq m_1$, and $Y_1(0) = D_1$ has full (column) rank.

On the other hand, if the Markov parameters of the combined system and of system 2 are known, then the Markov parameters of system 1 can be solved according to the following equations:

$$Y_1(0) = Y_t(0)Y_2^T(0)[Y_2(0)Y_2^T(0)]^{-1} \tag{44}$$

$$Y_2(i) = \left[Y_t(i) - \sum_{\tau=0}^{i-1} Y_1(\tau)Y_2(i - \tau)\right]Y_2^T(0)\ [Y_2(0)[Y_2^T(0)]^{-1} \qquad i = 1, 2, \ldots \tag{45}$$

provided $m_2 \geq q_2$, and $Y_2(0) = D_2$ has full (row) rank.

Numerical Examples

In this section, two numerical examples are used to illustrate the identification procedure for closed-loop system developed in this paper. Example 1 illustrates the simple case of identifying a system with an existing linear output feedback controller. Example 2 presents a case with a dynamic feedback controller. It is the purpose of the examples to outline step-by-step the series of computations involved for each of the respective closed-loop cases. A discussion regarding practical aspects of the algorithm is also provided.

Example 1

Consider a two-input, three-output, sixth-order system whose discrete system matrices are given as follows:

$$A = \begin{bmatrix} 0.9691 & 0.0154 & 0.0001 & 0.2120 & 0.0014 & 0.0000 \\ 0.0154 & 0.9690 & 0.0155 & 0.0014 & 0.2139 & 0.0014 \\ 0.0000 & 0.0077 & 0.9768 & 0.0000 & 0.0007 & 0.2127 \\ -0.2817 & 0.1395 & 0.0009 & 0.9458 & 0.0180 & 0.0001 \\ 0.1407 & -0.2838 & 0.1412 & 0.0180 & 0.9634 & 0.0182 \\ 0.0005 & 0.0699 & -0.2122 & 0.0000 & 0.0091 & 0.9544 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0232 & 0.0001 \\ 0.0001 & 0.0233 \\ 0.0000 & 0.0000 \\ 0.2120 & 0.0014 \\ 0.0014 & 0.2139 \\ 0.0000 & 0.0007 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Let the system be stabilized by a feedback controller $u(i) = Fy(i)$ where the feedback gain is given as

$$F = \begin{bmatrix} -0.375 & 0 & 0 \\ 0 & -0.500 & 0 \end{bmatrix}$$

For closed-loop identification, the closed-loop can be excited by a random perturbation of the system output $y(i)$, i.e., $u(i) = F[y(i) + y_e(i)]$. In this case, the excitation input is $v(i) = Fy_e(i)$. The excitation signal for the second output is shown in Fig. 2, and the resultant closed-loop system response for the second output is shown in Fig. 3.

From the closed-loop excitation data, the observer Markov parameters for the closed-loop system are identified. Since the true order of the system is six and the system has three outputs, the number of observer Markov parameters that can be identified is two or greater, i.e., $p \geq 2$. For $p = p_{\min} = 2$, the identified observer Markov parameters are listed as

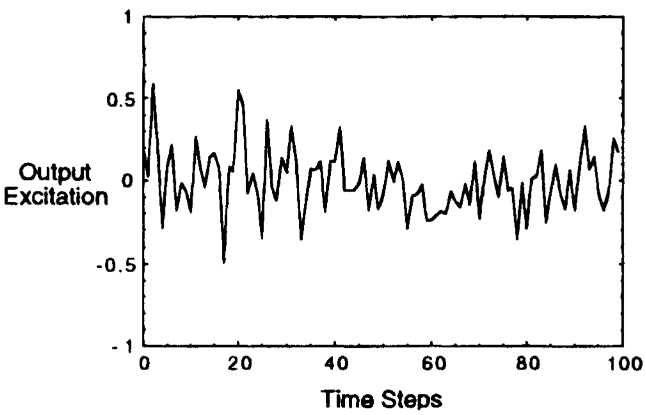

Fig. 2 Excitation signal for closed-loop identification.

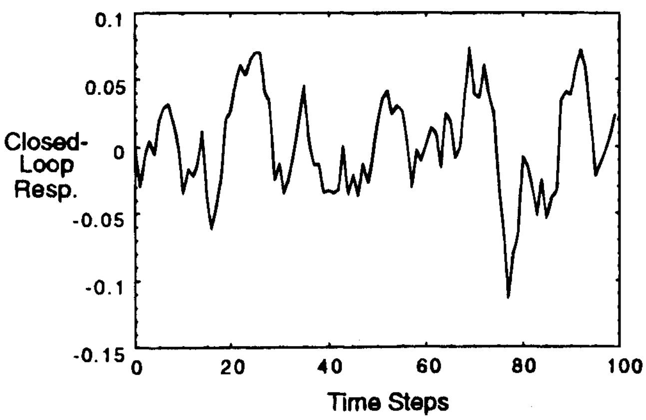

Fig. 3 Response of closed-loop system to excitation signal.

$$\bar{Y}_c(1) = \begin{bmatrix} 0.2120 & 0.0014 & 1.8354 & 0.0326 & 0.0001 \\ 0.0014 & 0.2139 & 0.0329 & 1.8254 & 0.0337 \\ 0.0000 & 0.0007 & 0.0001 & 0.0164 & 1.9312 \end{bmatrix}$$

$$\bar{Y}_c(2) = \begin{bmatrix} -0.2120 & -0.0014 & -0.8969 & -0.0020 & 0.0000 \\ -0.0014 & -0.2139 & -0.0021 & -0.8874 & -0.0029 \\ 0.0000 & -0.0007 & 0.0000 & -0.0011 & -0.9774 \end{bmatrix}$$

As shown in the theoretical development, the two identified observer Markov parameters completely describe the closed-loop system, from which any number of closed-loop and open-loop system Markov parameters can be computed. In the noise-free case as in this example, the Markov parameters can be recovered exactly. The calculation is illustrated here for the first few Markov parameters. The closed-loop Markov parameters $Y_c(1)$, $Y_c(2)$, $Y_c(3)$, ..., are computed using Eq. (14)

$$Y_c(1) = \bar{Y}_c^{(1)}(1) = \begin{bmatrix} 0.2120 & 0.0014 \\ 0.0014 & 0.2139 \\ 0.0000 & 0.0007 \end{bmatrix}$$

$$Y_c(2) = \bar{Y}_c^{(1)}(2) + \bar{Y}_c^{(2)}(1)Y_c(1) = \begin{bmatrix} 0.1771 & 0.0081 \\ 0.0081 & 0.1776 \\ 0.0000 & 0.0042 \end{bmatrix}$$

$$Y_c(3) = \bar{Y}_c^{(2)}(1)Y_c(2) + \bar{Y}_c^{(2)}(2)Y_c(1) = \begin{bmatrix} 0.1353 & 0.0190 \\ 0.0190 & 0.1330 \\ 0.0002 & 0.0100 \end{bmatrix}, \ldots$$

where $\bar{Y}_c^{(1)}(1)$, $\bar{Y}_c^{(2)}(1)$ and $\bar{Y}_c^{(1)}(2)$, $\bar{Y}_c^{(2)}(2)$ are the 3×2 and 3×3 partitions of $\bar{Y}_c(1)$ and $\bar{Y}_c(2)$, respectively. Beginning with $Y_c(3)$, the extra closed-loop Markov parameters are computed by setting $\bar{Y}_c(i) = 0, i = 3, 4, \ldots$. The open-loop system Markov parameters, $Y(1)$, $Y(2)$, $Y(3)$, ..., can be computed using Eq. (15).

$$Y(1) = Y_c(1) = \begin{bmatrix} 0.2120 & 0.0014 \\ 0.0014 & 0.2139 \\ 0.0000 & 0.0007 \end{bmatrix}$$

$$Y(2) = Y_c(2) - Y_c(1)FY(1) = \begin{bmatrix} 0.1940 & 0.0084 \\ 0.0084 & 0.1995 \\ 0.0000 & 0.0043 \end{bmatrix}$$

$$Y(3) = Y_c(3) - Y_c(1)FY(2) - Y_c(2)FY(1)$$
$$= \begin{bmatrix} 0.1648 & 0.0208 \\ 0.0208 & 0.1733 \\ 0.0002 & 0.0106 \end{bmatrix}, \ldots$$

In the same fashion, all desired closed-loop and open-loop system Markov parameters can be computed from the two identified observer Markov parameters. The same result is obtained if more observer Markov parameters are identified with the exception that the observer Markov parameters are now set to zero at a late time step. For example, say for $p = 3$, the observer Markov parameters $\bar{Y}_c(i)$ are set to zero for $i = 4, 5, \ldots$.

Shown in Fig. 4 is the closed-loop pulse response function of the second-output first-input pair computed from the two

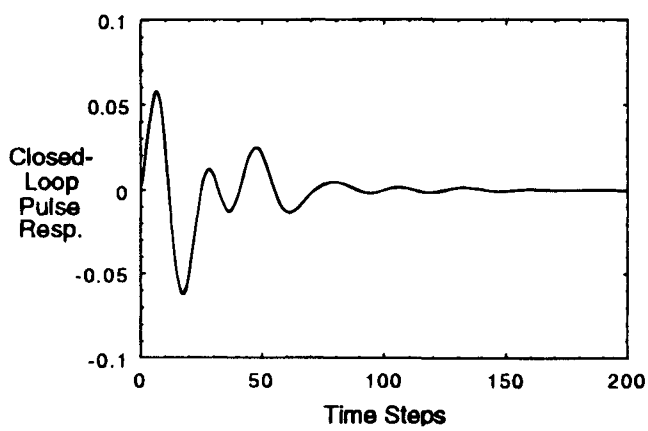

Fig. 4 Identified closed-loop pulse response (Closed-loop Markov parameters).

identified observer Markov parameters. Shown in Fig. 5 is the result obtained for the corresponding open-loop pulse response. Results for the other input-output pairs are similar and not shown here.

Example 2

This example illustrates the case of closed-loop identification with an existing closed-loop dynamic controller. This time, a direct transmission term is also included in the open-loop model,

$$D = \begin{bmatrix} 0.1165 & 0.0352 \\ 0.0627 & -0.0697 \\ 0.0075 & 0.1696 \end{bmatrix}$$

The controller matrices are given as follows:

$$P = \begin{bmatrix} 0.3392 & 0.0054 & 0.0000 & 0.0742 \\ 0.0054 & 0.3391 & 0.0054 & 0.0005 \\ 0.0000 & 0.0027 & 0.3419 & 0.0000 \\ -0.0986 & 0.0488 & 0.0003 & 0.3310 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.0654 & -0.0071 & -0.0024 \\ -0.0335 & -0.0708 & -0.0419 \\ 0.0303 & -0.0667 & -0.0402 \\ -0.0189 & 0.0517 & 0.0452 \end{bmatrix}$$

$$R = \begin{bmatrix} -0.0004 & 0.0031 & 0.0032 & 0.0030 \\ 0.0040 & 0.0060 & -0.0106 & 0.0089 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.0191 & -0.0072 & 0.0083 \\ -0.0624 & -0.0085 & -0.0134 \end{bmatrix}$$

The closed-loop system is excited by a random perturbation. The time histories of the excitation signal $v_1(i)$, $v_2(i)$, the system input $u(i)$, the system output $y(i)$, and the controller state $s(i)$, each of 100 data samples, are recorded for system identification purposes. Since the dimension of the augmented state is $n + n_s = 6 + 4 = 10$, and the dimension of the augmented output is $q + n_s = 3 + 4 = 7$, the minimum number of observer Markov parameters that are to be identified is two, $p_{\min} = 2$. For this value of p the identified observer Markov parameter matrix is of dimensions 7×32. From this matrix, any number of closed-loop Markov parameters for the augmented system can be computed. For illustration, the first few Markov parameters are computed next.

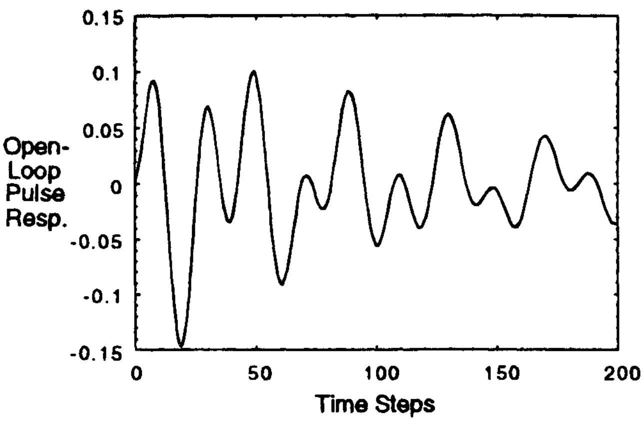

Fig. 5 Identified open-loop pulse response (Open-loop Markov parameters).

$$Y_{ac}(0) = \bar{Y}_{ac}(0) = D_a$$

$$= \begin{bmatrix} 0.1165 & 0.0352 & 0 & 0 & 0 & 0 \\ 0.0627 & -0.0697 & 0 & 0 & 0 & 0 \\ 0.0075 & 0.1696 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Y_{ac}(1) = \bar{Y}^{(1)}_{ac}(1) + \bar{Y}^{(2)}_{ac}(1)(I - D_aF_a)^{-1}Y_{ac}(0)$$

$$= \begin{bmatrix} 0.2124 & 0.0019 & 0 & 0 & 0 & 0 \\ -0.0003 & 0.2131 & 0 & 0 & 0 & 0 \\ -0.0000 & 0.0007 & 0 & 0 & 0 & 0 \\ 0.0072 & 0.0024 & 1 & 0 & 0 & 0 \\ -0.0087 & -0.0034 & 0 & 1 & 0 & 0 \\ -0.0009 & -0.0011 & 0 & 0 & 1 & 0 \\ 0.0014 & 0.0034 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the fact that $\bar{Y}_{ac}(i) \equiv 0$, $i = 3, 4, \ldots$, the remaining closed-loop Markov parameters of the augmented system can be computed. For example,

$$Y_{ac}(2) = \bar{Y}^{(1)}_{ac}(2) + \bar{Y}^{(2)}_{ac}(1)(I - D_aF_a)^{-1}Y_{ac}(1) + \bar{Y}^{(2)}_{ac}(2)(I - D_aF_a)^{-1}Y_{ac}(0)$$

$$Y_{ac}(3) = \bar{Y}^{(2)}_{ac}(1)(I - D_aF_a)^{-1}Y_c(2) + \bar{Y}^{(2)}_{ac}(2)(I - D_aF_a)^{-1}Y_{ac}(1)$$

Finally, the Markov parameters $Y_a(i)$ of the augmented system are recovered.

$$Y_a(1) = Y_{ac}(1)(I - F_aD_a) = \begin{bmatrix} 0.2120 & 0.0014 & 0 & 0 & 0 & 0 \\ 0.0014 & 0.2139 & 0 & 0 & 0 & 0 \\ 0.0000 & 0.0007 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Y_a(2) = Y_{ac}(2)(I - F_aD_a) - Y_{ac}(1)F_aY_a(1)$$

$$= \begin{bmatrix} 0.1940 & 0.0084 & 0 & 0 & 0 & 0 \\ 0.0084 & 0.1995 & 0 & 0 & 0 & 0 \\ 0.0000 & 0.0043 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The upper left partitions of $\bar{Y}_{ac}(0)$, $Y_a(1)$, $Y_a(2), \ldots$, are exactly the open-loop system Markov parameters $D, CB, CAB, \ldots$.

Discussion

In the ideal case where the system is linear and the data is noise free, it is indeed possible to identify the open-loop system exactly from closed-loop data as shown in the examples. An important feature of the proposed approach is the introduction of deadbeat observers into the time-domain identification equations. One can write an exact relationship between the input and output data in terms of a finite number of (closed-loop) observer Markov parameters. The observer gain is not to be designed or known a priori, but rather it is specified implicitly via a deadbeat condition. The observer Markov parameters beyond p are required to be identically zero, where p denotes the number of observer Markov parameters to be identified. For a linear system, as long as p is chosen such that p times the number of outputs is at least equal to the order of the system then it is possible to identify the open-loop system exactly by this method. Thus for a large order system, p need not be large if the system has several outputs. The observer, in a sense, provides a mechanism to compress the parameter space if so desired. In reality, however, since all systems have noises and nonlinearities, a larger value of p tends to produce more accurate results. This is observed in the examples when noises are added to the data. Qualitatively speaking, this is because the noises are allowed to smooth out when p is chosen to be larger than the minimum value. For a detailed treatment of plant and measurement noises by the observer-based approach, the readers are referred to Refs. 1, 2, and 4. Furthermore, since the open-loop system is recovered from the closed-loop system, accuracy in the identification of the closed-loop system becomes very important when noises are present. This accuracy is influenced by not only the method used but also by the richness and the duration of the excitation. As with practically all system identification methods, the results must be examined on a case-by-case basis. Finally, in terms of numerical stability, the most critical part of the algorithm is the least-squares solution. Routinely, this is handled by the well-known singular value composition which is quite robust numerically.

Concluding Remarks

This paper develops a procedure for identification of open-loop systems operating in the closed loop. First, the basic problem of identifying the open-loop system with an existing linear output feedback controller is formulated. The approach used here is to inject additive excitation during closed-loop operation, and closed-loop data is used for identification. The procedure consists of two basic steps. First, through an associated observer, the closed-loop system Markov parameters are identified. Second, the open-loop system is recovered from the identified closed-loop parameters. Relations between the identified observer Markov parameters, and the closed-loop system Markov parameters are established. The considerably more complicated case of closed-loop identification of a system with an existing dynamic feedback controller is also treated in this paper. The developed solution requires perturbation of both the input signal and the controller state for identification. Under this condition, it is shown that the identification problem with an existing dynamic controller can be formulated to mimic the problem of identification with an linear output feedback controller of an augmented dynamic system, which has a simple closed-form solution. For the case of two cascading linear systems, it is sometimes possible to recover the Markov parameters of one system when the Markov parameters of the other system and those of the combined system are known. Numerical examples are provided to illustrate the computational procedure of the proposed approach.

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